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Asymptotic Conformal Invariance of SU(2) and Standard Models in Curved Space-time

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Abstract

The asymptotic conformal invariance of some SU(2) model and Standard Model in curved space-time are investigated. We have examined the conditions for asymptotic conformal invariance for these models numerically.

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I. INTRODUCTION

The asymptotic behavior of quantum field theory(QFT) in curved space-time is quite important in the early Universe considerations. The typical action for the arbitrary renormalizable theory(including scalar sector) in curved space-time has the following form [1];

$$S = \int d^n x \sqrt{-g} (L_{matter} + \frac{1}{2} \xi R \varphi^2 + L_{ext}) \tag{1}$$

where L_{matter} is the same Lagrangian as typical for flat space-time (with changes $\partial_\mu \to \nabla_\mu, \gamma^\mu \to \gamma^\mu(x)$), and L_{ext} is the Lagrangian for external fields [2]. It is quite well-known that the asymptotic behavior of an arbitrary QFT is best investigated through renormalization group(RG) where coupling constants are getting effective coupling constants.

In the action Eq.(1) the new scalar-gravitational coupling constant ξ appears (if compared with flat space-time). The asymptotic behavior of this effective coupling in non-abelian asymptotically free gauge theories has been first investigated in Ref. [2]. In these works, Buchbinder and Odintsov has shown that at high energies (strong curvature) the beautiful phenomenon, asymptotic conformal invariance, may be realized. It means that $\xi(t) \to \frac{1}{6}$ as $t \to \infty$ where t is RG parameter, *i.e.* in matter sector theory is trying to become conformally invariant at high energies. (Later on, the other types of $\xi(t)$ behavior have been found [2,3] like $\mid \xi(t) \mid \to \infty$ or $\xi(t) = \xi$ for a theory [1–4])

The value of ξ is important in realization of different types of inflationary Universe. For example, for some inflationary models ξ should be very large.

In the present letter, we study the behavior of $\xi(t)$ in SU(2) gauge theories with spinors and scalars introduced in Ref. [5] in flat space-time. However, we study its behavior not in asymptotically free regime on special solutions of RG as it has been done in Ref. [1,2] but in general solutions (hence, numerically). We find that the theory is still asymptotically conformally invariant. In the section 3, we discuss the RG behavior of $\xi(t)$ numerically in the Standard Model(SM). It is found that SM does not have asymptotic conformal invariance in general except for a special case, $\xi_0 = \frac{1}{6}$.

II. SU(2) MODEL IN CURVED SPACE-TIME

In curved space-time, the SU(2) gauge theory with scalars and spinors [1] had been investigated when the gauge coupling g, Yukawa coupling h and quartic scalar coupling f are asymptotically free on special solutions of RG equations [5]. We have investigated the behavior of ξ in general.

The form of the SU(2) gauge theory in curved space-time is [1]

$$\begin{aligned} S &= \int d^n x \sqrt{-g}[L_{YM} + \sum_{k=1}^{m} i\psi^a_{(k)}\gamma^\mu(x) D^{ab}_\mu \psi^b_{(k)} \\ &+ \frac{1}{2}(D^{ab}_\mu \varphi_b)^2 - ih\epsilon^{acb}\psi^a_{(k)}\psi^b_{(k)}\varphi_c - \frac{f}{4!}(\varphi^2_a)^2 + \frac{1}{2}\xi R\varphi^2], \\ L_{YM} &= -\frac{1}{4}(\nabla_\mu A^a_\nu - \nabla_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\nu)^2. \end{aligned} \tag{2}$$

This action contains scalars φ^a, spinors ψ^a_k belonging to adjoint representation of the gauge group and gauge fields A^a_μ; $a = (1, 2, 3)$, $k = (1, 2, 3, ..., m)$, m is the number of spinor multiplets. D^{ab}_μ is the general covariant derivative, and it is defined as follows;

$$\begin{aligned} D^{ab}_\mu \varphi_b &= \nabla_\mu \delta^{ab}\varphi_b + ig\epsilon^{acb}A^c_\mu \varphi_b, \\ D^{ab}_\mu \psi_b &= \nabla_\mu \delta^{ab}\psi_b + ig\epsilon^{acb}A^c_\mu \psi_b + \frac{1}{2}\omega^{\alpha\beta}_\mu \sigma_{\alpha\beta}\psi_b \end{aligned} \tag{3}$$

where the matrices $\sigma_{\alpha\beta}$ are given by the relation

$$\sigma_{\alpha\beta} = \frac{1}{2}(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha),$$

and $\omega^{\alpha\beta}_\mu$ is the spinor connection which satisfies the torsionless condition;

$$\partial_\mu e^\alpha_\nu - \partial_\nu e^\alpha_\mu + e_{\mu\beta}\omega^{\alpha\beta}_\nu - e_{\nu\beta}\omega^{\alpha\beta}_\mu = 0. \tag{4}$$

From the above, following four equations are obtained using the Schwinger-De Witt technique [1,6];

$$(4\pi)^2\frac{dg^2}{dt} = -(14-\frac{16}{3}m)g^4, \tag{5}$$

$$(4\pi)^2\frac{dh^2}{dt} = 16h^4 - 24g^2h^2, \tag{6}$$

$$(4\pi)^2\frac{df}{dt} = \frac{11}{3}f^2 - 24g^2f + 72g^4 + 16fh^2 - 96h^4, \tag{7}$$

$$(4\pi)^2\frac{d\xi}{dt} = (\xi-\frac{1}{6})(\frac{5}{3}f + 8h^2 - 12g^2). \tag{8}$$

The special solutions of $g^2(t)$, $h^2(t)$ and $f^2(t)$ are [5]

$$g^2(t) = \frac{g_0^2}{1+cg_0^2t}, \quad c \equiv \frac{1}{16\pi^2}(14-\frac{16}{3}m), \tag{9}$$

$$h^2(t) = (\frac{5}{8}+\frac{1}{3}m)g^2(t), \tag{10}$$

$$f^2(t) = (\frac{120}{11}m + \frac{32}{11}m^2 - \frac{207}{22})g^4(t). \tag{11}$$

These solutions are asymptotically free for $m = 1, 2$.

With the above asymptotically free solutions, the solution of ξ is

$$\xi = (\xi_0 - \frac{1}{6})(1+cg_0^2t)^{\frac{B}{c}} + \frac{1}{6}, \tag{12}$$

where

$$B \equiv \frac{1}{16\pi^2}[\frac{5}{3}\sqrt{\frac{120}{11}m + \frac{32}{11}m^2 - \frac{207}{22}} + 8(\frac{5}{8}+\frac{1}{3}m) - 12]. \tag{13}$$

For $m = 1$, $\xi \to \frac{1}{6}$ as $t \to \infty$ ($\frac{B}{c} \approx -0.38 < 0$).

For $m = 2$, $\xi \to \infty$ as $t \to \infty$ ($\frac{B}{c} \approx 7.58 > 0$).

This case has been investigated in Ref. [2] already.

However we investigated the RG-solution of ξ for the general solutions of the other couplings (h and f). The general solution of the $h^2(t)$ is

$$h^2(t) = g^2(t)\frac{(g_0^2)^{-\frac{a}{c}}h_0^2a\pi^2}{(g_0^2)^{-\frac{a}{c}}h_0^2 + (g^2(t))^{-\frac{a}{c}}[a\pi^2g_0^2 - h_0^2]}, \tag{14}$$

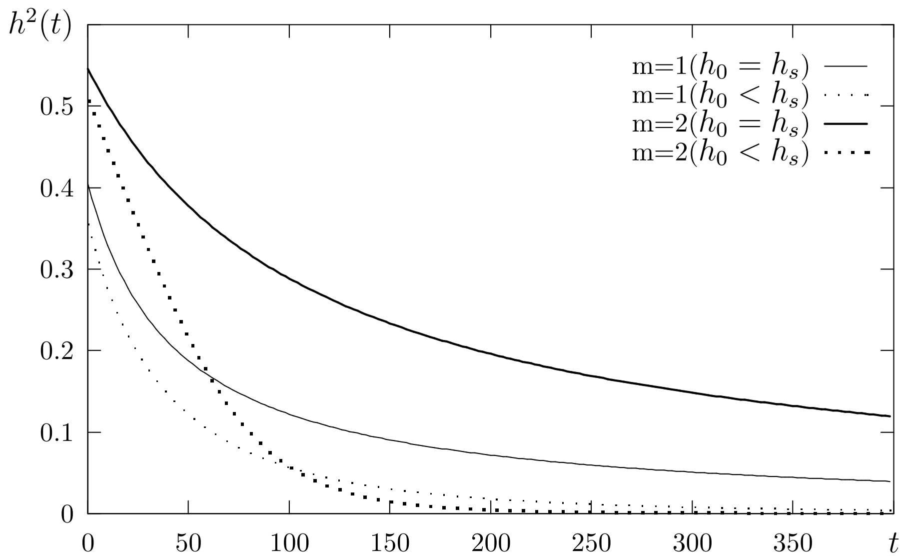
$$a \equiv \frac{1}{\pi^2}(\frac{5}{8}+\frac{1}{3}m).$$

It is the unique solution(Eq.(6) can be linearized). The $h^2(t)$ has asymptotically free solution only if $m = 1, 2$. For $m \geq 3$, the $h^2(t)$ becomes negative. The initial value of h_0 is crucial. Due to the positivity condition, only for $h_0^2 \leq (\frac{5}{8} + \frac{1}{3}m)g_0^2 \equiv h_s^2$, the solution is meaningful. In the case of equality, the general solution is reduced to the special solution of Eq.(10) [Fig.1].

The solution of f has two types. If the solution of $h^2(t)$ is the special one(*i.e.* $h_0^2 = h_s^2$), we have the analytical solution;

$$f(t) = g^2(t)A\frac{Q_0 + A + \lambda(Q_0 - A)}{Q_0 + A - \lambda(Q_0 - A)}, \tag{15}$$

where $\quad Q_0 \equiv \frac{f_0}{g_0^2}, \quad A \equiv \sqrt{\frac{24}{11}(5m + \frac{4}{3}m^2 - \frac{69}{16})}, \quad \lambda \equiv (1 + cg_0^2 t)^{\frac{11A}{24\pi^2 c}}.$

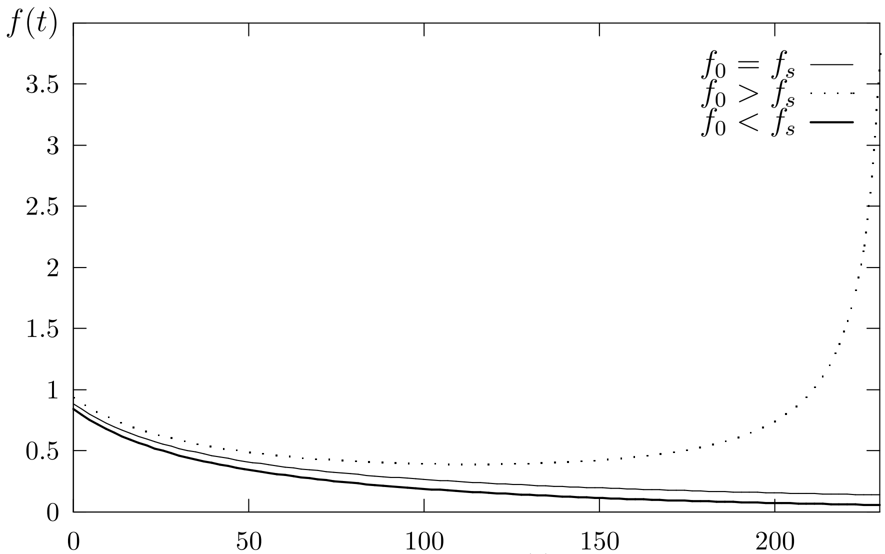


[Fig.1]. Plot of the Yukawa coupling having its initial value h_s and h_0 smaller than h_s.

The solution becomes the special solution Eq.(10) for $Q_0 = A$, *i.e.* $f_0 = Ag_0^2 \equiv f_s$. For $Q_0 > A$, (*i.e.* $f_0 > f_s$), the solution has a singularity. For $m = 1$ and 2, $\lambda \to \infty$ as $t \to \infty$ because c and $\frac{A}{c}$ are positive. In this case, we have an approximate solution of $f(t)$ as follow;

$$f(t) \approx -g^2(t)\sqrt{\frac{24}{11}(5m+\frac{4}{3}m^2-\frac{69}{16})}, \tag{16}$$

which is negative even though very small at large t [Fig.2]. In the case of $h_0^2 = h_s^2$, $f(t)$ changes its behavior near $f_0 = f_s$ sensitively. For $h_0^2 < h_s^2$, it is very hard to deal with the Eq.(7) analytically. From the definition of a and c, $\frac{a}{c} = -\frac{23}{13}$ for $m = 1$ and $\frac{a}{c} = -\frac{31}{5}$ for $m = 2$.



[Fig.2]. Plot of quartic potential coupling $f(t)$ for its initial value f_0, $h_0 = h_s$.

Therefore $h^2(t)$ decreases much faster than $g^2(t)$ for $m = 1, 2$. Then we can ignore h terms at large t. Thus, Eq.(7) becomes

$$(4\pi)^2\frac{df}{dt} = \frac{11}{3}f^2 - 24g^2f + 72g^4. \tag{17}$$

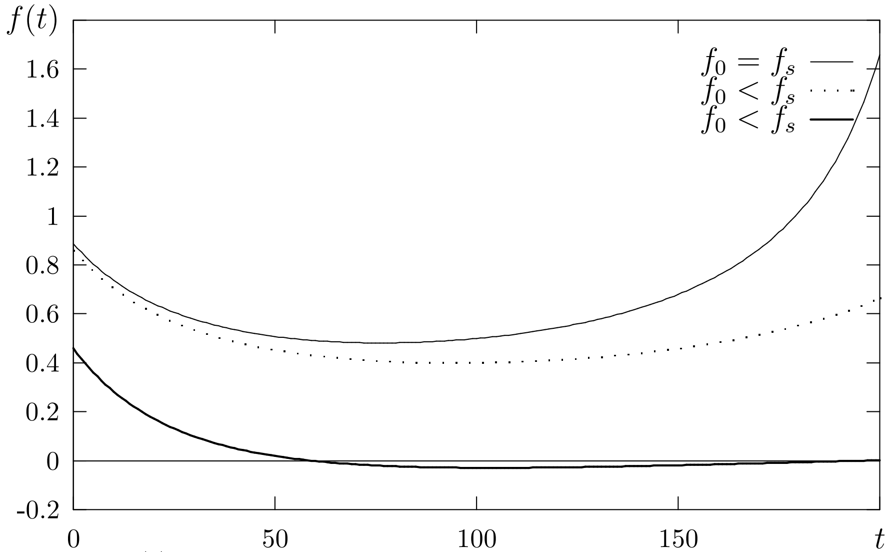
The solution is

$$f(t) = \frac{24\pi^2}{11}\beta g^2(t)\frac{\beta\tan[\frac{\beta}{2c}\ln(1+cg_0^2t)] + \frac{11}{24\pi^2}\frac{f_0}{g_0^2} - a}{\beta - \tan[\frac{\beta}{2c}\ln(1+cg_0^2t)](\frac{11}{24\pi^2}\frac{f_0}{g_0^2} - a)} + \frac{24\pi^2}{11}ag^2(t), \tag{18}$$

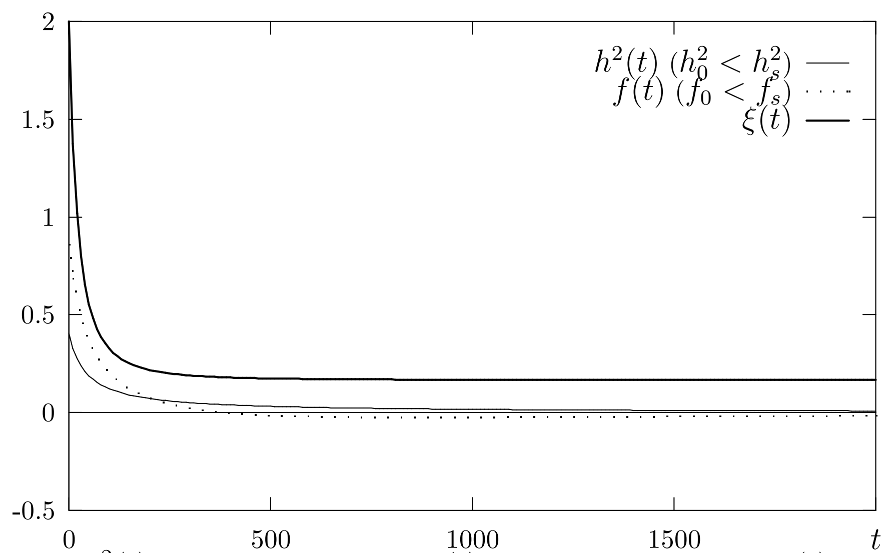
where $\quad \beta \equiv \sqrt{\frac{1}{\pi^4}(-\frac{1}{9}m^2 - \frac{10}{24}m + \frac{239}{64})}.$

For $f_0 > f_s$ and $f_0 < f_s$ at $h_0^2 < h_s^2$, it is difficult to manipulate the equations analytically. Thus, numerical solution is investigated for this case. The initial g_0 is given by $g_0 = 0.650$

[7]. In the case of $h_0^2 < h_s^2$, the $f(t)$ are similar to a parabola independent of choice of f_0 [Fig.3].



[Fig.3]. Plot of $f(t)$ having its initial value f_s, f_0 smaller than f_s and more smaller than f_s



[Fig.4]. $h^2(t)$ is asymptotically free. $f(t)$ becomes small negative. $\xi(t)$ goes to $\frac{1}{6}$.

The solution of ξ in the case of $h_0^2 = h_s^2$, $f_0 \neq f_s$ at large t is

$$\xi = (\xi_0 - \frac{1}{6})(1 + cg_0^2 t)^{\frac{D}{c}} + \frac{1}{6}, \tag{19}$$

where

$$D \equiv \frac{1}{16\pi^2}[-\frac{5}{3}\sqrt{\frac{24}{11}(\frac{69}{16}+5m+\frac{4}{3}m^2)}+8(\frac{5}{8}+\frac{1}{3}m)-12]. \tag{20}$$

For $m=1$ and 2, $\xi \to \frac{1}{6}$ as $t \to \infty$, because $\frac{D}{c}$ is negative [Fig.4]. But the potential gets unstable because $f(t)$ becomes negative at large t. For $h_0^2 < h_s^2$, we can ignore $h^2(t)$ for large t in Eq.(8). Then, Eq.(8) becomes

$$(4\pi)^2\frac{d\xi}{dt} = (\xi - \frac{1}{6})(\frac{5}{3}f - 12g^2). \tag{21}$$

Then, the solution of ξ for large t is

$$\xi = (\xi_0 - \frac{1}{6})\exp[\ln[\cos(\theta+\kappa)^{\frac{20}{11}}] + \ln[(1+cg_0^2t)^{-\frac{1}{c}(\frac{6}{11}a+\frac{3}{\pi^2})}] + const] + \frac{1}{6}, \tag{22}$$

where

$$\theta \equiv \tfrac{\beta}{2c}\ln(1+cg_0^2t), \quad \kappa \equiv \arctan[\tfrac{11}{24\pi^2}\tfrac{f_0}{g_0^2}+a].$$

Therefore, $\xi \to \frac{1}{6}$, when $t \to \infty$ for $m = 1, 2$.

III. STANDARD MODEL IN CURVED SPACE-TIME

Nowadays, the Standard Model(SM) is considered as the most believable theory. We may ask whether the SM has the asymptotic conformal invariance in curved space-time! We have investigated the question through the SM one-loop RG-equations.

We choose a gauge the 'tHooft-Landau gauge. In this gauge the, $W^\pm$, Z and photon are transverse, and the associate ghosts are massless and couple only to the gauge fields; the would be goldstone bosons $G^\pm, G$ have a common mass driving from the scalar potential only. Moreover, the gauge parameter is not renormalized in this gauge, so it does not enter into the RG equations [7,8].

The effective potential $V(\phi)$ through one-loop is [7,8]

$$\begin{aligned}V(\phi) &= \Omega'(\mu, m^2, h, g, g') + \frac{1}{2}m^2\phi^2 + \frac{1}{24}\lambda\phi^4 \\ &+ \kappa[\frac{1}{4}H^2(\ln\frac{H}{\mu^2} - \frac{3}{2}) + \frac{3}{4}G^2(\ln\frac{G}{\mu^2} - \frac{3}{2}) + \frac{3}{2}W^2(\ln\frac{W}{\mu^2} - \frac{5}{6}) \\ &+ \frac{3}{4}Z^2(\ln\frac{Z}{\mu^2} - \frac{5}{6}) - 3T^2(\ln\frac{T}{\mu^2} - \frac{3}{2})] + ...,.\end{aligned} \tag{23}$$

where

$$\kappa = 16\pi^2, \quad H = m^2 + \frac{1}{2}\lambda\phi^2, \tag{24}$$
$$T = \tfrac{1}{2}h^2\phi^2,\, G = m^2 + \frac{1}{6}\lambda\phi^2,$$
$$W = \tfrac{1}{4}g^2\phi^2,\, Z = \frac{1}{4}(g^2 + g'^2)\phi^2.$$

The SM one-loop RG-equations are [7,8]

$$16\pi^2\frac{dg^2}{dt} = -\frac{19}{3}g^4, \tag{25}$$
$$16\pi^2\frac{dg'^2}{dt} = \frac{41}{3}g'^4, \tag{26}$$
$$16\pi^2\frac{dg_3^2}{dt} = -14g_3^4, \tag{27}$$
$$16\pi^2\frac{dh^2}{dt} = 9h^4 - 16g_3^2h^2 - \frac{9}{2}g^2h^2 - \frac{17}{6}g'^2h^2, \tag{28}$$
$$16\pi^2\frac{d\lambda}{dt} = 4\lambda^2 + 12\lambda h^2 - 36h^4 - 9\lambda g^2 - 3\lambda g'^2 + \frac{9}{4}g'^4 + \frac{9}{2}g^2g'^2 + \frac{27}{4}g^4, \tag{29}$$
$$16\pi^2\frac{d\xi}{dt} = (\xi - \frac{1}{6})(2\lambda + 6h^2 - \frac{9}{2}g^2 - \frac{3}{2}g'^2), \tag{30}$$

where g', g, g_3, h, λ, and ξ are U(1), SU(2), SU(3), top quark Yukawa (other Yukawa couplings are ignored), quartic scalar, and non-minimal couplings respectively. Especially, β-functions for gauge couplings are [8]

$$\beta_{g'} = \frac{5}{3}g'^3(\frac{4}{3}n + \frac{1}{10}), \quad \beta_g = g^3(\frac{4}{3}n - \frac{43}{6}), \tag{31}$$

where n is the number of generation ($n = 3$ yields the above results). The equation for ξ is obtained using the technique of Ref. [1].

At $\mu = M_z$, the initial values are [7]

$g_0 = 0.650$,

$g'_0 = 0.358,$

$\alpha_3 = 0.10, 0.11, 0.12, 0.13,$

$h_0 = 1.17,$

$\lambda_0 = \lambda_0,$

$\xi_0 = \xi_0,$

where we have used the top quark mass $188GeV$ in deciding the initial value of $h(t)$ [9].

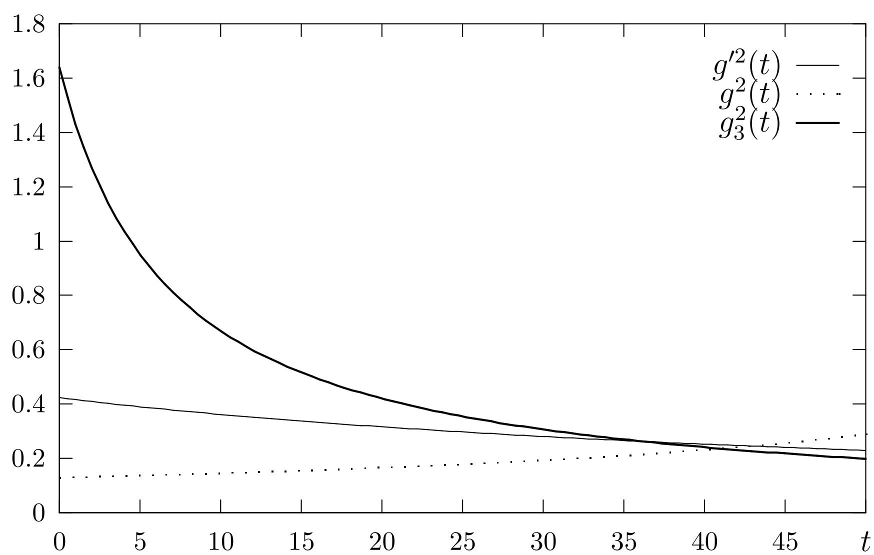
The analytical solution for U(1), SU(2), and SU(3) couplings can be easily found as follow;

$$g'^2(t) = \frac{g'^2_0}{1 - \frac{41}{48\pi^2} g'^2_0 t}, \tag{32}$$

$$g^2(t) = \frac{g^2_0}{1 + \frac{19}{48\pi^2} g^2_0 t}, \tag{33}$$

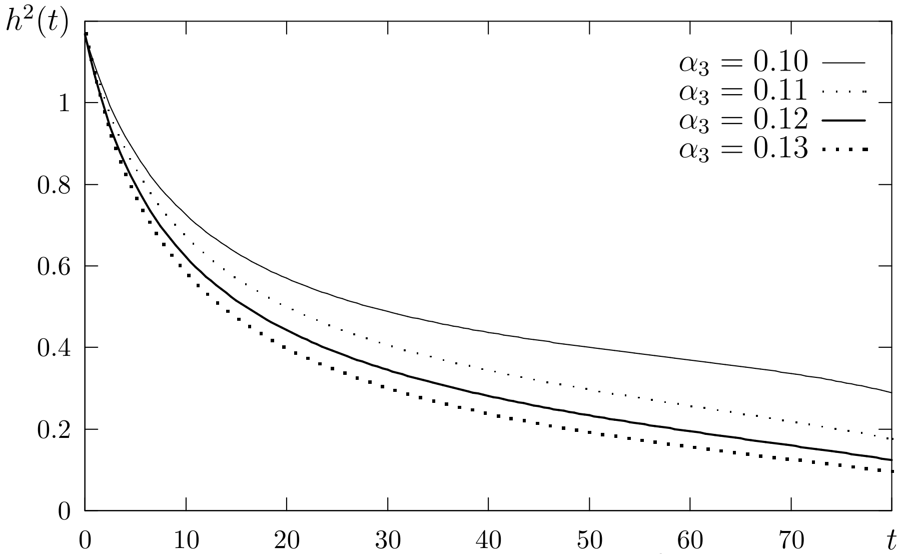
$$g_3^2(t) = \frac{g^2_{30}}{1 + \frac{7}{8\pi^2} g^2_{30} t}. \tag{34}$$

Because the other equations seem to be impossible to have analytical solutions, numerical solutions are investigated.

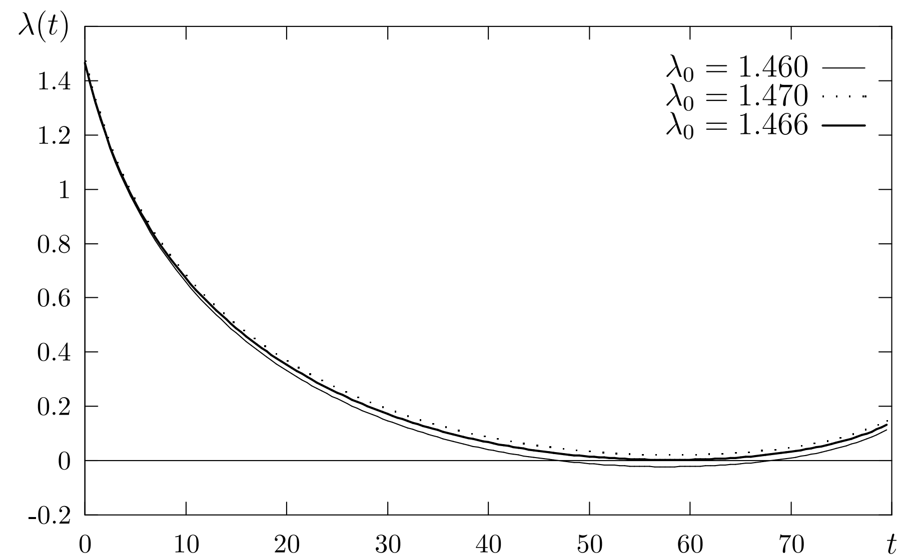


[Fig.5]. Plot of the U(1), SU(2) and SU(3) gauge couplings at α_3=0.13.

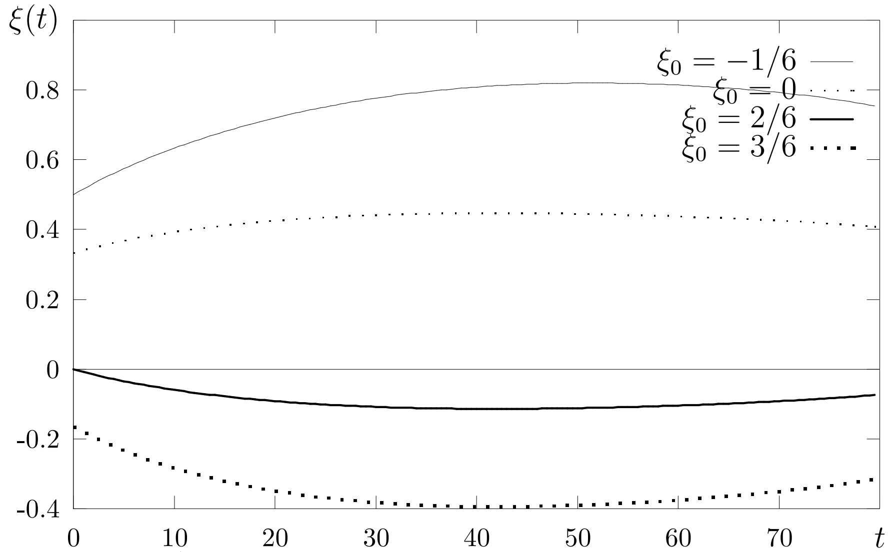
Fig.5 is the graph of gauge couplings for $\alpha_3 = 0.13$. Increasing the input value of α_3 causes h to decrease faster as t increases [Fig.6].



[Fig.6]. Plot of the top quark Yukawa coupling h at m_t =188GeV.



[Fig.7]. Plot of the running coupling λ for m_t=188GeV and α_3=0.13.



[Fig.8]. Plot of the ξ with its initial value -1/6, 0, 2/6, 3/6 for m_t=188GeV and α_3=0.13.

The quartic potential coupling λ is very sensitive to it's initial value [Fig.7]. There is a critical λ_0 such that $\lambda(t)$ is positive at all scale for $\lambda_0 \geq \lambda_{crit}$, which means the stability of electroweak vacuum. Requiring the stability of the electroweak vacuum results in the lower limit on M_H [7,10,11]. In this analysis, we found the mass of the Higgs as $M_H \geq 178 GeV$ at $\alpha_3 = 0.13$.

The behavior of $\xi(t)$ for SM is similar to a shooting motion [Fig.8]. Standard Model does not have asymptotic conformal invariance at high energy except the case $\xi_0 = 1/6$. However, some other extensions of SM may have asymptotic conformal invariance. Moreover, if we consider a grand unified model at high energy, some GUT models might have the asymptotic conformal invariance compared with the results of Ref. [1] where SU(N) asymptotically conformal invariant GUT models have been found.

IV. CONCLUSION

In some model of the SU(2) gauge theory with scalars and spinors, there exists the special initial value condition of h and f, which makes all couplings asymptotically free. In the

case of the general solution of RG-equations of couplings, it is found that the theory has still asymptotic conformal invariance. In Standard Model, it is found that ξ does not approach to $\frac{1}{6}$ asymptotically in general. However, if Standard Model has asymptotic conformal invariance, ξ should be $\frac{1}{6}$ at all scale (it's unlikely). Even though there is no asymptotic conformal invariance in Standard Model generally, but it may happen that other extensions of Standard Model or some GUT models may have it [12,13].

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